Relaxation of Stochastic Dominance Constraints via Optimal Mass Transport

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Abstract

Stochastic orders define comparisons between random variables and play a prominent role in statistics, economics, and insurance due to their connection to risk-averse preferences. Furthermore, many generalizations and extensions to vector-valued outcomes and stochastic processes are available (see, e.g., [3]).

Stochastic dominance relations were introduced into stochastic optimization in [1]. In this framework, a benchmark random outcome with a desired distribution is selected, and the random outcomes resulting from decisions are compared to it via stochastic dominance.

Optimization problems with stochastic dominance constraints have attracted considerable attention because they provide a way to shape the risk associated with the optimal decision very precisely. We refer to [2] for an overview of the theory and methods of optimization under stochastic dominance constraints.

While in many practical situations natural benchmarks are available, other applications may lack such options. In such a scenario, an ideal benchmark may be selected that is not necessarily achievable but is still desirable. In this case, it remains of great interest to determine how to approach the set of distributions that dominate the ideal benchmark.

This talk addresses the problem of choosing a tight relaxation of the stochastic dominance constraint by selecting a feasible distribution that is closest to those dominating the benchmark in terms of mass transportation distance. The theory of optimal mass transport (see [4]) provides various metrics to quantify the distance between distributions and has recently gained significant attention in the context of data science.

In our presentation, we focus on second-order stochastic dominance in a standard atomless space. We present explicit formulas for the Monge–Kantorovich transportation distance between a given distribution and the set of dominating distributions. Under an additional assumption, we construct the associated projection of the distribution of interest onto the set of distributions dominating the benchmark. We also present a numerical method for solving the relaxation problem. The method is implemented and tested on a disaster management problem, and our numerical results illustrate the efficiency of the proposed approach.

Further extensions will be presented time-permiting.

References

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